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Correction to "Hamiltonian Hopf Bifurcation with Symmetry"

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Ian Melbourne has pointed out to us that the statement and the proof of Theorem 3.4 of our paper do not match, the current formulation being empty of content. We thank him for drawing our attention to this. The correct statement of Theorem 3.4 is given below in the notation of the original paper. We also provide the minor modification of the proof required by this new formulation.

Let $V_0 \subset V$ be the vector subspace defined in (2.11) and $G^\xi := \{g \in G \mid \mathrm{Ad}_g \xi = \xi\}$. Let n^ξ denote the minimum number of geometrically distinct relative equilibria for an arbitrary $G^\xi \times S^1$-equivariant vector field on the unit sphere in V_0 (with respect to the norm in Lemma 2.1). Two relative equilibria z_1 and z_2 are *geometrically distinct* when they are not in the same $G^\xi \times S^1$ orbit. This definition extends trivially to relative periodic orbits.

Theorem 3.4. *Let* (V, ω, h_λ) *be a one-parameter family of G-Hamiltonian systems that satisy conditions* (H1)–(H4). *Let H be a closed subgroup of G with Lie algebra* $\mathfrak{h}$. *Then, for each* $\xi \in \mathfrak{h}$ *close to zero, each energy level close to zero, and all* λ *near* $\lambda_\circ$, *there are at least* n^ξ *geometrically distinct relative periodic orbits of relative period close to* $2\pi/\nu_0$.

Proof. The modification of the original proof relies on the following simple observation. Let ∇g be a gradient vector field on the vector space V with respect to the inner product $\langle \cdot, \cdot \rangle$. If H is a Lie group acting on V by isometries leaving the potential function g invariant, then any relative equilibrium of ∇g is actually an equilibrium. Indeed, if $v \in V$ is a relative equilibrium of V, then there exists an element $\xi \in \mathfrak{h}$ in the Lie algebra $\mathfrak{h}$ of H such that $\nabla g(v) = \xi \cdot v$. At the same time $\|\nabla g(v)\|^2 = \langle \nabla g(v), \nabla g(v) \rangle = \langle \nabla g(v), \xi \cdot v \rangle = \mathbf{d}g(v) \cdot (\xi \cdot v) = 0$ by the invariance properties of g. This implies that $\nabla g(v) = 0$ and that $v \in V$ is therefore an equilibrium of ∇g.

The proof provided in the paper is valid up to the last line, where we have to show that $G^\xi \times S^1$-relative equilibria (and not just equilibria) of the vector field on the sphere given by the function in (3.10) are solutions of the reduced bifurcation equation in (3.5) and hence RPOs of the original system.

The group $G^\xi \times S^1$ acts by isometries on V_0 and leaves the potential of the gradient vector field $B(v_0, \alpha, \lambda, \xi)$ (Lemma 3.7) invariant. By the remark above, any $G^\xi \times S^1$-relative equilibrium of this function considered as a vector field in V_0 is an equilibrium, for any value of the parameters (α, λ, ξ). Now, a relative equilibrium of the vector field on the sphere defined by the function in (3.10) amounts to a relative equilibrium of $B(v_0, \alpha, \lambda, \xi)$ for the value of the parameter λ given by the function $\lambda(r, u_0, \alpha, \xi)$. The argument of the original proof involving the gradient nature of B, shows that this relative equilibrium is a solution of the bifurcation equation and consequently an RPO of the original system. □

If dim $H = 0$, the theorem gives periodic orbits and we recover the result of VAN DER MEER [vdM85].
On page 17, in line 6 of Corollary 3.10, the word "relative" is missing in front of "equilibria".
On page 18, in line 8 of the proof of Corollary 3.12, the word "zeros" should be replaced by "relative equilibria".

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